Physics 200B **Mechanics II** **Winter 2013**

Problem Set I: Due Thursday, January 24, 2013

1.) Derive Hamilton's equations directly from a modified version of the Principle of Least Action.

2.) Construct the Hamiltonian for the heavy symmetric top with one point fixed and derive the Hamiltonian equations of motion. Assume the top spins in a gravitational field.

3.) Fetter and Walecka 6.4

4.) Consider an acoustic wave propagating in a medium with index of refraction $n(\underline{x})$, in the limit of geometrical optics.

a.) Show that the incremental total phase $d\Phi$ can be written as:

$$d\Phi = k \cdot dx - \omega dt .$$

Derive the ray equations by extremizing, Φ.

b.) Extremize the 'abbreviated phase' $\Phi_0 = \int k d\ell$ to obtain a differential equation for the ray path. Explain the difference between this result and that of (a.).

c.) Simplify your result from (b.) and carefully discuss its physical content. Give examples in your discussion.

5.) Consider the Helmholtz equation for a sound wave in a medium with index of refraction $n(\underline{x})$.

$$\nabla^2 \psi + \frac{\omega^2}{c_0^2} n(\underline{x})^2 \psi = 0 .$$

a.) For $n(\underline{x})^2 = 1 + \delta(\underline{x})$, where $\delta << 1$, and assuming sound is beamed in the $\hat{z}$ direction, show the Helmholtz equation may be (approximately) re-written as:

$$2i k_z \frac{\partial \psi}{\partial z} + \nabla_\perp^2 \psi + \frac{\omega^2}{c_0^2} \delta(\underline{x}) \psi = 0 .$$

b.) Define k_r here. The above equation is called the parabolic wave equation. Discuss

i.) the approximations inherent to this formulation.

ii.) the physical meaning of the different terms.

iii.) the restrictions on $\partial\psi/\partial z$, etc.

c.) Now, write $\psi = A(\underline{x})e^{i\phi(\underline{x})}$. Use the parabolic wave equation to derive coupled equations for phase $\phi(\underline{x})$ and amplitude $A(\underline{x})$. Discuss the physical content of your result. Can you relate your result to that obtained using eikonal theory?

6.) Consider a harmonic oscillator with distinct spring constants for the x, y, z directions. Derive and solve the Hamilton-Jacobi equation for this system.

7.) Consider an ocean with sound speed a function of depth, so that $c_s(z)$ is maximal at z_0, where z is depth as measured from the surface. Using Fermat's Principle, determine the path that a ray takes to traverse a long distance ℓ.